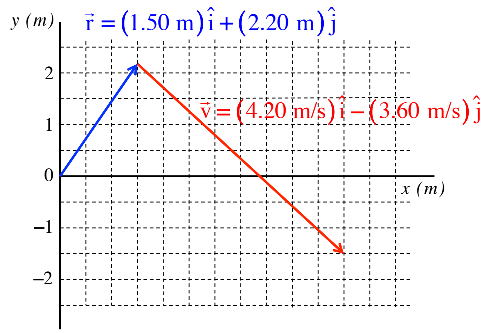
Problem 11.12

This problem is designed to point out a very odd characteristic about *angular momentum*, that a body moving in a *straight line* can have *angular momentum*! (I'll show why, on a conceptual level, this is the case at the end.) Following the math:



$$\vec{L} = \vec{r} \times [m \vec{v}]$$

$$= \left[(1.50\text{ m})\hat{i} + (2.20\text{ m})\hat{j}\right] \times \left[(1.50\text{ kg})\left[(4.20\text{ m/s})\hat{i} - (3.60\text{ m/s})\hat{j}\right]\right]$$

There are two ways to do this. One is to treat the vectors like *unit vectors* and use the matrix approach. The other is to convert to *polar notation* and the do the cross product that way. I'll do the latter first, then the former.

Using the polar approach:

$$|\vec{L}| = |\vec{r} \times \vec{p}|$$

$$= \left|\left[(1.50\text{ m})\hat{i} + (2.20\text{ m})\hat{j}\right] \times \left[(1.50\text{ kg})\left[(4.20\text{ m/s})\hat{i} - (3.60\text{ m/s})\hat{j}\right]\right]\right|$$

$$= \left[\left((1.50\text{ m})^2 + (2.20\text{ m})^2\right)^{1/2} \measuredangle \tan^{-1}\left(\frac{2.20\text{ m}}{1.50\text{ m}}\right)\right] \times \left[(1.50\text{ kg})\left[\left((4.20\text{ m/s})^2 + (-3.60\text{ m/s})^2\right)^{1/2}\right] \measuredangle \tan^{-1}\left(\frac{-3.60\text{ m}}{4.20\text{ m}}\right)\right]$$

$$= \left[(2.66\text{ m}) \measuredangle 55.7^\circ\right] \times \left[(8.30\text{ kg} \cdot \text{m/s}) \measuredangle -40.6^\circ\right]$$

$$= (2.66\text{ m})(8.30\text{ kg} \cdot \text{m/s})\sin\left(55.7^\circ - (-40.6^\circ)\right)$$

$$= 21.9\text{ kg} \cdot \text{m}^2/\text{s}$$

Using the right-hand rule (or the fact that the rotation of clockwise), we get a negative angular momentum so that

$$\vec{L} = -\left(21.9\text{ kg} \cdot \text{m}^2/\text{s}\right)\hat{k}$$

Using the polar approach:

$$|\vec{r}| = \left[\left((1.50\text{ m})^2 + (2.20\text{ m})^2\right)^{1/2} \measuredangle \tan^{-1}\left(\frac{2.20\text{ m}}{1.50\text{ m}}\right)\right]$$

$$= \left[(2.66\text{ m}) \measuredangle 55.7^\circ\right]$$

$$|\vec{p}| = m \quad \vec{v}$$

$$= (1.50\text{ kg})\left((4.20\text{ m/s})^2 + (-3.60\text{ m/s})^2\right)^{1/2} \measuredangle \tan^{-1}\left(\frac{-3.60\text{ m}}{4.20\text{ m}}\right)$$

$$= (8.30\text{ kg} \cdot \text{m/s}) \measuredangle -40.6^\circ$$

$$|\vec{L}| = |\vec{r}| \quad |\vec{p}| \quad \sin \quad \phi$$

$$= (2.66\text{ m})(8.30\text{ kg} \cdot \text{m/s})\sin\left(55.7^\circ - (-40.6^\circ)\right)$$

$$= 21.9\text{ kg} \cdot \text{m}^2/\text{s}$$

Using the right-hand rule (or the fact that the rotation of clockwise), we get a negative *angular momentum* so that

$$\vec{L} = -\left(21.9\text{ kg} \cdot \text{m}^2/\text{s}\right)\hat{k}$$

Using the unit vector approach:

$$\vec{L} = \vec{r} \times \vec{p}$$

$$= \left[(1.50\text{ m})\hat{i} + (2.20\text{ m})\hat{j}\right] \times \left[(1.50\text{ kg})\left[(4.20\text{ m/s})\hat{i} - (3.60\text{ m/s})\hat{j}\right]\right]$$

$$= \left[(1.50\text{ m})\hat{i} + (2.20\text{ m})\hat{j}\right] \times \left[(6.30\text{ kg} \cdot \text{m/s})\hat{i} - (5.40\text{ kg} \cdot \text{m/s})\hat{j}\right]$$

with a matrix solution of:

$$\vec{L} = \vec{r} \times \vec{p} = \left|\begin{array}{ccc|cc} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ 1.5 & 2.2 & 0 & 1.5 & 2.2 \\ 6.3 & -5.4 & 0 & 6.3 & -5.4 \end{array}\right.$$

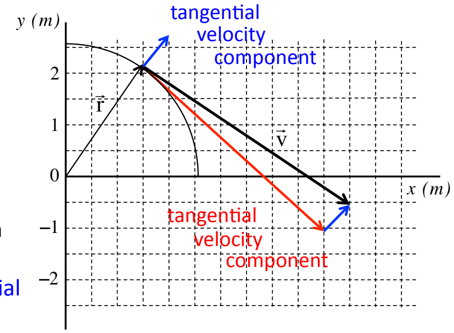
$$= (\hat{k})\left[(1.5)(-5.4) - (2.2)(6.3)\right]$$

$$= -\left(22.0\text{ kg} \cdot \text{m}^2/\text{s}\right)\hat{k}$$

Close enough for government work!

You are done.

EXTRA: Conceptually (i.e., aside from the fact that the math maintains that this is so), how can a body moving in a straight line have *angular momentum*?



The *velocity vector* in the sketch has a radial component and a tangential component. The radial component is associated with motion radially toward or away from the origin, whereas the tangential component is associated with motion that CIRCLES the origin (at least instantaneously). It is that instantaneous circular motion that has *angular momentum* associated with it.